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Hotelling T^2 Approach in Analysing the Results from Systematic Design

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ABSTRACT

The present investigation was undertaken with a view to examine relative utility of T^2 statistic in relation to analysis of variance for analysing the data collected from Fan design; a systematic field layout. Cotton crop was used for the purpose. Fan design (FAN) gave correlated error within block and heterogeneous error variances, limiting use of analysis of variance. T^2 approach showed significant differences among spacing means observed in Fan design. Similar differences were also observed for the same spacing treatments in randomised block design (RBD), indicating that the results in Fan are in line with the results in RBD. Therefore T^2 statistic can be used for analysing the data collected from systematic layouts like FAN.

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INTRODUCTION

The agricultural experiments are conducted in the conventional designs like randomised block design, split plot design etc. The spacing experiment is a special group of experiments which need large area under each replication for testing a number of spacing treatments in the conventional designs. Since the area required is large, the cost of the experiment increases. Any alternative to the conventional design with about the same efficiency and less operative cost would be preferred in the present time when costs of inputs have tremendously increased, and land available for experimentation has become a limiting factor. Nelder (1962) advocate the use of systematic design (hereafter termed as Fan Design) for the large number of spacing treatments, compacted in a much smaller area as compared to the conventional designs.

The spacing treatments are arranged in single row plots in sequence without randomization in fan design. Therefore, the errors of the adjacent plots in fan design are correlated within a block. It is also likely that the error variances may differ from treatment to treatment. (Greenberg, 1951; Salmon, 1955). In such conditions, the method of analysis of variance is not valid for testing

$$H_O : \mu_1=\mu_2=\ldots=\mu_K$$

Any method(s) of analysing data of fan design, will be of much useful in the field of agricultural experimentation.

Experimental :

The field experiment was conducted using fan and randomized block designs

during the years 1973-74 and 1974-75, on cotton variety IAN 579-188 with a view to assess the relative utility of Hotelling T^2 in relation to analysis of variance for analysing the data. The treatments consisted various spacings which varied from 0.045 sq.m. to 0.270 sq.m. per plant in both the lay outs. The number of bolls per plant and cotton yield data were collected and subjected to statistical analysis.

Hotelling T^2 Approach :

Graybill (1954) has described the procedure developed by Hsu (1938) to test $H_o : \mu_1=\mu_2=\ldots=\mu_K$ when errors are heterogeneous and correlated within a block but are independent from block to block. The errors are likely to be heterogeneous and correlated within block of the fan design, hence T^2 statistic could be used for anlysing the data under the assumptions that (i) the block effect is random and (ii) fertility gradient is absent (very small compact area is needed per block in fan design and hence there is little chance of fertility gradient within block).

The mathematical model for the jth response in i^{th} block is $X_{ij}=\mu+B_i+t_j+e_{ij}$

$(i = 1,2,\ldots..,b; j = 1,2,\ldots.,K.)$

where, μ = general mean; B_i = effect of ith block;

t_j = effect of jth spacing; e_{ij} = error associated with j^{th} spacing in i^{th} block.

e_{ij} is assumed to be normally distributed with mean zero and variance σ^2 and such that the correlation between e_{ij} and e_{km} is zero if $i \neq k$.

The test of $H_o : \mu_1 = \mu_2=\ldots.=\mu k$ can be carried out by Hotelling T^2 statistic computed from the mean vector and coveriance matrix of the differences.

$Y_{ij} = X_{ij} - X_{i, j+1}$

of the observations on the adjacent responses.

Let $\bar{Y}'$ be the $k-1$ component vector of differences of successive response means $(\bar{Y}_1, \bar{Y}_2, \ldots. \bar{Y}_{k-1}) = (\bar{X}_1-\bar{X}_2, \bar{X}_2-\bar{X}_3 \ldots., X_{k-1}-\bar{X}_k)$ and S_y be the $(k-1) \times (k-1)$ covariance matrix of the Y_{ij} differences.

Then

$$T^2 = b.\ \bar{Y}'.\ S_{\bar{Y}}^{-1}.\ \bar{Y}$$

and $$F = \frac{b-k+1}{(b-1)\ (k-1)}\ T^2$$

has the F distribution with $k-1$ and $b-k+1$ degrees of freedom. If the level of significance is α, the H_o is rejected if

$F \geqslant F\alpha,\ k-1,\ b-k+1$

RESULTS AND DISCUSSION

The intraclass correlations were worked out for both the attributes measured in fan design. The pooled estimates for these attributes are 0.62 and 0.74 respectively and are significant. Therefore the errors are correlated in fan design.

The χ^2 values were also calculated by Bartlett's test to test the homogeneity of error variances. The data in fan design were analysed by method of analysis of variance without any consideration about error and by Hotelling T^2 method. The results are presented in Table 1 along with the F values for randomised block design (RBD).

The χ^2 value for the number of bolls per plant as well as cotton yield per plant were significant indicating the heterogeneity in error variances. The F values (FAN) calculated by analysis of variance method were significant. These F values were large which could be either due to the real differences among the spacing means or consequence of the correlated errors and or heterogeneity of error vaiances. The F values (FAN) worked out by T^2 approach

TABLE 1

χ^2 and F values for number of bolls per plant and cotton yield per plant.

Attribute	*Year*	χ^2	*F value*		
			FAN		*RBD*
			Analysis of variance approach	*T^2 approach*	*Analysis of variance approach*
Number of bolls per plant	1973-74	50.22**	52.47**	18.70**	5.14*
	1974-75	35.83**	38.66**	14.85**	3.78*
Cotton yield per plant	1973-74	59.28**	28.31**	13.52*	18.04**
	1974-75	36.93**	14.14**	9.84*	12.22**

* and ** indicate significant value at 5% and 1% levels respectively.

showed significant differences among the spacing means for the number of bolls per plant as well as cotton yield per plant in both the years. Therefore, the treatment means differed from eachother. Similar differences were also observed in RBD as evident from the F-values (RBD). The results in FAN are in line with the results in RBD for both the attributes.

The results, therefore, reveal that the Hotelling T^2 method may be used to analyse the data collected from FAN design, when experimental site has uniform fertility. The results obtained by T^2 approach in fan design are similar to those obtained in RBD. The use of analysis of variance for the analysis of the data in systematic layout does not give the real picture of the differences in treatment means when the errors are correlated. The assumption about independent errors is also not fulfilled in fan design. Therefore, the method of analysis of variance should not be used for fan design. As against this T^2 approach has given similar results as in RBD for both the attributes. This leads to the conclusion that wherever errors are correlated (like error in fan design) within block and or the error variances are heterogeneous, the T^2 approach should be used for anlysing the data.

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